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Resonance Blocking of Traveling Waves by a System of Cracks in an Elastic Layer

E. V. Glushkov[a], N. V. Glushkova[a], M. V. Golub[a], and Ch. Zhang[b]

[a] *Kuban State University, ul. Stavropol'skaya 149, Krasnodar, 350040 Russia*
[b] *University Siegen, Paul-Bonarz str. 9–11, 57076 Germany*
e-mail: evg@math.kubsu.ru

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Abstract—Wave processes that occur in an elastic layer when waves traveling in it are diffracted by a system of horizontal cracks are investigated. Integral representations of wave fields are constructed in terms of the convolution of Green's matrices and unknown jumps of displacements at the cracks. The displacement jumps are determined from the boundary integral equations, which are obtained from the initial boundary-value problem with the boundary conditions at crack faces being satisfied. The spectrum of the integral operator is studied for different variants of mutual crack arrangement and is compared with the spectrum of the corresponding operators for individual cracks; the relationship between the spectrum and the blocking effects is analyzed. The possibility of obtaining an extended frequency band of waveguide blocking in the case of groups of cracks is demonstrated.

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INTRODUCTION

Diffraction of elastic waves by internal obstacles (cracks, cavities, inclusions) and resonance effects observed in the diffracted field are of interest in the development of wave methods for location and identification of defects and in problems of the strength and fracture of materials and structures. In addition, the character of propagation and blocking of traveling waves is of special interest. For example, the phenomenon of abrupt screening of signals by systems of periodic obstacles (such as systems of interdigital contacts, grooves, etc.) is used in acoustoelectronics to create frequency filters based on surface acoustic waves [1]. Dielectric structures (photonic crystals) with wide band gaps are used in photonics [2, 3]. Similar band gaps are characteristic of the propagation of acoustic waves (phonons) in periodic composite and crystal structures (atomic phonon lattices) [4, 5]. Analogous effects are observed in elastic media with periodic sets of cracks [6], which are close to the subject of this paper.

The question arises as to whether sufficiently wide band gaps can be formed in elastic waveguides without the use of a large number of (periodic) obstacles. In particular, whether the trapped mode effect, which is well-known for waveguides with obstacles [7–10] and consists in the capture of incoming wave energy and its localization in the vicinity of the obstacle, can be used for this purpose. This effect is closely related to spectral properties of the corresponding boundary-value problem. It occurs at frequencies ω close to the complex frequencies of natural vibrations ω_n of the waveguide with the defect under consideration (ω_n is the spectral point of the boundary-value problem). Eigensolutions $\mathbf{u}_n$ corresponding to spectral points ω_n describe the wave field localization in the vicinity of the defect. One of the manifestations of the effect of trapping is an abrupt screening (filtration) of the corresponding frequency components of a transient signal propagating along the waveguide. If the obstacle is represented by a crack, the resonance blocking is usually accompanied by a sharp increase in the dynamic stress intensity at the crack front.

Previously, phenomena of resonance capture and blocking were considered by the examples of an elastic waveguide with an obstacle in the form of a single horizontal [11, 12] or inclined [13] cut (crack). On the basis of analytic-computational solution of the corresponding diffraction problems, we analyzed the distribution of spectral points ω_n in the complex frequency plane as a function of crack size and position in the waveguide; we also analyzed their effect on the frequency characteristics of signal propagation and reflection. In particular, we showed that resonance poles ω_n fall on the real axis for certain geometric parameters; i.e., we confirmed the possibility of the appearance of a real point of a discrete spectrum in the continuous spectrum of traveling waves [14] and constructed the corresponding eigensolutions that describe the wave field of the trapped mode.

On the other hand, resonance blocking occurs in a very narrow frequency band near $\omega \approx \mathrm{Re}\,\omega_n$, which degenerates into a point for $\mathrm{Im}\,\omega_n \longrightarrow 0$. One can expect to extend the band of almost total waveguide blocking if several discrete spectral points lying close to each other and to the real axis appear owing to the appearance of additional obstacles.

This paper continues our earlier studies described in [11, 12]. As the first step, we analyze the changes in spectrum structure and in the blocking properties caused by the interference of adjacent cracks. Different crack arrangements are considered: cracks in one plane, cracks at different heights, cracks spaced along the waveguide, and stacked cracks (one over another). We determine how wave interaction between the cracks changes the frequencies ω_n and how these changes affect the blocking properties. In particular, we determine how much the results for the simpler case of a single crack can be useful for analyzing the blocking properties of a system of cracks. The possibility of extending the frequency band of resonance blocking of a waveguide by a special choice of the system of several cracks is discussed.

MATHEMATICAL MODEL

As was mentioned, the phenomenon of resonance capture and localization of wave energy may accompany diffraction by obstacles of different nature. For definiteness and comparability with earlier results, we consider the obstacles in the form of thin horizontal cuts (cracks). Therefore, the mathematical statement and the general scheme of solution remain the same as those given in [11, 12]; the notation is also retained wherever possible.

In the framework of the two-dimensional statement of the problem, we consider an elastic waveguide (layer) that occupies the strip $|x| < \infty$, $-H < z < 0$ in the Cartesian coordinate system xOz. The layer has several cuts (cracks) of zero-valued thickness Ω_m: $|x - x_m| < l_m$, $z = -d_m$ (see Fig. 1). Complex amplitudes $\mathbf{u} = \{u_x, u_z\}$ of steady-state harmonic vibrations of layer particles $\mathbf{u}e^{-i\omega t}$ with angular frequency ω satisfy the Lamé equations for an isotropic elastic medium; the harmonic factor $e^{-i\omega t}$ will be omitted in what follows.

The layer boundaries $z = 0$ and $z = -H$ are free of stresses: $\boldsymbol{\tau} = \{\tau_{xz}, \sigma_z\} = 0$, excluding the region where the load $\mathbf{q}_0$ generating the initial wave field $\mathbf{u}_0$ is applied. In what follows, we take $\mathbf{u}_0$ to be one of the generated modes, namely, the mode determined by the contribution of the residue from the real pole of Green's matrix $\zeta \equiv \zeta_k$: $\mathbf{u}_0 = \mathbf{a}_0 e^{i\zeta x}$ (see Eq. (2.8) in [11]). In this case, the specific form of the load $\mathbf{q}_0$ is insignificant and we only assume that it is applied at a sufficient distance, to the left of the system of cracks $\Omega = \bigcup \Omega_m$.

We assume that the surfaces of cuts do not contact being free of stresses, $\boldsymbol{\tau}|_\Omega = 0$. The displacement

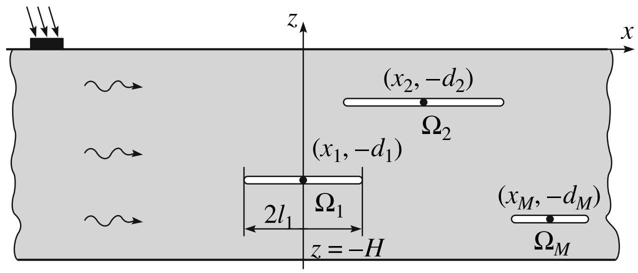

Fig. 1. Elastic waveguide with a system of horizontal cracks.

field $\mathbf{u}$ at the cuts is discontinuous with unknown jumps:

$$\mathbf{v}_m(x) = \mathbf{u}(x, -d_m - 0) - \mathbf{u}(x, -d_m + 0),$$
$$|x - x_m| < l_m, \quad m = 1 \ldots M.$$

As distinct from the case of a single crack, the diffracted wave field $\mathbf{u}_C$ is now the sum of M fields $\mathbf{u}_m$ induced by individual jumps $\mathbf{v}_m$. Hence, the combined field $\mathbf{u}$ formed in the waveguide has the form

$$\mathbf{u} = \mathbf{u}_0 + \sum_{m=1}^{M} \mathbf{u}_m,$$

where $\mathbf{u}_0$ is the given field of the source or one of the incoming waves; the fields $\mathbf{u}_m$ and the corresponding stress fields $\boldsymbol{\tau}_m$ are given by the former integral representations (see Eq. (2.6) in [11]):

$$\mathbf{u}_m(x, z) = \frac{1}{2\pi}\int_\Gamma N_m(\alpha, z)\mathbf{V}_m(\alpha)e^{-i\alpha x}d\alpha,$$
$$\boldsymbol{\tau}_m(x, z) = \frac{1}{2\pi}\int_\Gamma S_m(\alpha, z)\mathbf{V}_m(\alpha)e^{-i\alpha x}d\alpha, \qquad (1)$$

where $\mathbf{V}_m = F[\mathbf{v}_m]$ are the Fourier transforms (symbols) of the unknown displacement jumps at the cuts. It should be noted that the specific form of the symbols of Green's matrices N_m and S_m depends on the crack depth d_m, so that these matrices are different for cracks lying in different planes.

In the case of a single crack, the substitution of the integral representation of the stress field $\boldsymbol{\tau}$ into the boundary conditions at the crack faces reduces the problem to the Wiener–Hopf integral equation with hypersingular matrix operator $\mathcal{L}$ in the unknown jump $\mathbf{v}$ (see Eq. (4.6) in [11]). In the case of a system of cracks, we introduce the generalized vector of unknown jumps $\mathbf{v} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_M\}$ and obtain the system of integral equations in this vector:

$$\mathcal{L}_m\mathbf{v}_m + \sum_{j=1, j\neq m}^{M} \mathcal{S}_{mj}\mathbf{v}_j = \mathbf{f}_m,$$

$$(x, z) \in \Omega_m, \quad m = 1 \ldots M,$$

$$\mathcal{L}_m \mathbf{v}_m \equiv \int\limits_{|x - x_m| < l_m} l_m(x-\xi)\mathbf{v}_m(\xi)d\xi$$

$$= \frac{1}{2\pi}\int\limits_{\Gamma} L_m(\alpha)\mathbf{V}_m(\alpha)e^{-i\alpha x}d\alpha, \tag{2}$$

$$\mathcal{S}_{mj}\mathbf{v}_j \equiv \int\limits_{|x - x_m| < l_m} s_{mj}(x-\xi)\mathbf{v}_j(\xi)d\xi$$

$$= \frac{1}{2\pi}\int\limits_{\Gamma} S_j(\alpha, -d_m)\mathbf{V}_m(\alpha)e^{-i\alpha x}d\alpha,$$

$$L_m(\alpha) = S_m(\alpha, -d_m),$$

$$\mathbf{f}_m = -\boldsymbol{\tau}_0(x, -d_m).$$

Each of the operators $\mathcal{L}_m$ describes the stress field that occurs within the segment Ω_m and corresponds to the displacement field $\mathbf{v}_m$ within the same segment without taking into account the presence of neighboring cracks. In essence, this is the same operator $\mathcal{L}$ with a hypersingular kernel $l(x-\xi)$ and a matrix kernel symbol $L(\alpha)$ as the operator in the problem of a single crack in the layer (see Eq. (4.4) in [11]). In turn, operators $\mathcal{S}_{mj}$ describe the stress fields in the region Ω_m caused by the jumps $\mathbf{v}_j$ within other segments Ω_j, $j \neq m$; i.e., they determine mutual influence of cracks. Because the points (x, z) at which the stress is determined do not belong to the region Ω_m at which the jump is given, the kernels of these operators $s_{mj}(x-\xi)$ are nonsingular.

The scheme that was used in [11] to reduce integral equations to an infinite algebraic system can be extended to the case of several regions Ω_m. However, in the latter case, the scheme becomes rather cumbersome. For this reason, we apply the Galerkin procedure combined with the expansion of the unknown jumps in the orthogonal Chebyshev polynomials of the second kind $U_k(x)$ with the weight $\sqrt{1-x^2}$, i.e.,

$$\mathbf{v}_m(x) = \sum_{k=1}^{N_m}\mathbf{c}_m^k p_m^k(x),$$

$$p_m^k(x) = U_k\left(\frac{x-x_m}{l_m}\right)\sqrt{l_m^2-(x-x_m)^2}, \tag{3}$$

and the use of projection along the same coordinate functions $p_m^l(x)$. As a result, we obtain a linear algebraic system of equations in unknown coefficients $\mathbf{c}_m^k(x)$:

$$\sum_{k=1}^{N_m} a_m^{lk}\mathbf{c}_m^k + \sum_{\substack{j=1\\ j\neq m}}^{M}\sum_{k=1}^{N_j} b_{mj}^{lk}\mathbf{c}_m^k = g_m^1,$$
$$m = 1, 2, \ldots, M, \tag{4}$$

which can be expressed in matrix form as follows:

$$D\mathbf{c} = \mathbf{g}. \tag{5}$$

Here, $\mathbf{c} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_M\}$ is the generalized vector of unknown expansion coefficients, where $\mathbf{c}_m = \{\mathbf{c}_m^1, \mathbf{c}_m^2, \ldots, \mathbf{c}_m^{N_m}\}$ are the vectors of the coefficients corresponding to individual jumps $\mathbf{v}_m$; the total length of the generalized vector (the dimension of matrix D) is $N = 2(N_1 + N_2 + \ldots + N_M)$.

The vector of the right-hand parts $\mathbf{g}$ is formed in a similar way in terms of projections $\mathbf{g}_m^l$ of the given field $\boldsymbol{\tau}_0$:

$$\mathbf{g}_m^l = \int\limits_{|x-x_m|<l_m} \boldsymbol{\tau}_0(x, -d_m)p_m^l(x)dx.$$

In the matrix

$$D = \begin{pmatrix} A_1 & B_{12} & \ldots & B_{1M} \\ B_{21} & A_2 & \ldots & B_{2M} \\ . & . & \ldots & . \\ B_{M1} & B_{M2} & \ldots & A_M \end{pmatrix},$$

the elements of diagonal blocks A_m of dimension $2N_m \times 2N_m$ and the elements of nondiagonal blocks B_{mj} of dimension $2N_m \times 2N_j$ have the following forms:

$$a_m^{lk} = \frac{1}{2\pi}\int\limits_{\Gamma} L_m(\alpha)P_k(\alpha l_m)P_l(-\alpha l_m)d\alpha,$$
$$l, k = 1, 2, \ldots, N_m,$$

$$b_{mj}^{lk} = \frac{1}{2\pi}\int\limits_{\Gamma} S_j(\alpha, -d_m)P_k(\alpha l_j)P_l(-\alpha l_m)e^{-i\alpha(x_j - x_m)}d\alpha,$$
$$l = 1, 2, \ldots, N_m, \quad k = 1, 2, \ldots, N_j,$$

where the functions $P_k(\alpha l_m) = J_{k+1}(\alpha l_m)/\alpha$ are the results of the Fourier transformation of the coordinate functions $F[p_k^m] = P_k(\alpha l_m)e^{i\alpha l_m}$.

With the coefficients $\mathbf{c}_m^k$ being calculated, the wave fields and their energy characteristics (the time-averaged energy flux density) are determined through representations (1) (by numerical integration) in the near-field zone and with the use of residue expansions (see

Eqs. (2.7) and (2.8) in [11]) in the far-field zone. To characterize the propagation of individual modes $\mathbf{u}_0(x, z) = \mathbf{a}_k e^{i\zeta_k x}$, we introduce, as earlier, the transmission and reflection coefficients $\kappa^{\pm} = E^{\pm}/E_0$, where E_0 is the energy of the incident wave $\mathbf{u}_0$ and E^+ and E^- are the energies of waves transmitted through the crack region and reflected from it, respectively. The quantities E_0 and $E_0^{\pm}$ are determined by numerical integration of the horizontal component of the corresponding energy density vectors over the waveguide cross section. The numerical solution is verified by the energy balance formula: $\kappa^+ + \kappa^- = 1$.

The eigenfrequencies ω_n (the poles of the harmonic wave field $\mathbf{u}(x, z, \omega)$ in the complex frequency plane ω) are approximated by the roots of the characteristic equation

$$\Delta(\omega) \equiv \det D(\omega) = 0. \qquad (6)$$

At the same time, the roots ω_n^m of the characteristic equations $\det A_m(\omega) = 0$ (for the diagonal matrix blocks of system D) in fact represent the resonance poles of the corresponding single individual cracks in the layer.

BLOCKING PROPERTIES OF THE SYSTEM OF CRACKS

At first glance, one can expect that, in the case when spacing between the regions Ω_m is sufficiently large, the spectrum ω_n will be the sum of spectra ω_n^m for each individual cut. On the other hand, the amplitudes of traveling waves $\mathbf{a}_k(z) e^{\pm i\zeta_k x}$ that ensure the wave interaction between the cracks are independent of x (for real-valued ζ_k) in the two-dimensional model under consideration. This means that the mutual effect must be the same for arbitrary crack-to-crack distances.

Finally, from the physical point of view, if the first obstacle has blocked the signal propagation, the presence of the following ones must not considerably affect the pattern, because the signal does not arrive at them. In other words, additional obstacles positioned behind the first one should not change the frequency of resonance blocking and, consequently, the positions of resonance poles that are close to the real axis.

Even the first calculations showed that wave interaction between obstacles realizes both seemingly alternative possibilities. The positions of the poles ω_n significantly vary with varying horizontal distance between the cracks; however, this occurs in such a way that resonance blocking frequencies conditioned by each individual crack remain intact.

Below, all the results are given in dimensionless form, which assumes a layer thickness H, a transverse wave velocity v_S, and shear modulus μ equal to unity. In this case, the dimensionless angular frequency is $\omega = 2\pi f H/v_S$, where f is the dimensional frequency in hertz. For definiteness, in all examples given below, we used Poisson's ratio $\nu = 1/3$ (for which $v_P/v_S = 2$, where v_P is the velocity of longitudinal waves in the layer) and $\mathbf{u}_0$ represented by the first (fundamental) antisymmetric mode a_0 [15] corresponding to the maximum wave number ζ_1 (the maximum real-valued pole of the Fourier symbol of the Green's function of the layer). This mode is most sensitive to the defects considered here [16].

Two Cracks in One Plane

Consider the incidence of mode a_0 on two cracks of unit halfwidth ($l_1 = l_2 = 1$) located at identical depths ($d_1 = d_2 = 0.25$). The distance between the crack centers $\Delta x_{12} = |x_1 - x_2|$ varies, and we assume that $x_1 = 0$, so that $\Delta x_{12} = x_2 > 2$.

In Fig. 2a, with the use of the grayscale pattern and level lines, we show the transmission factor κ^+ as a function of two parameters: distance x_2 and frequency ω. The darkest areas in the (x_2, ω) plane correspond to total blocking $\kappa^+ = 0$, and the white areas, to total transmission $\kappa^+ = 1$; the shades of gray give intermediate values of $\kappa^+(x_2, \omega)$. The solid lines $\omega = \operatorname{Re}\omega_n(x_2)$ imposed on the pattern show the real part of the resonance poles ω_n versus x_2 (the number of the line corresponds to the number n of the pole). The imaginary part $\omega = \operatorname{Im}\omega_n(x_2)$ is given in the lower plot by the solid lines with respective numbers (Fig. 2b).

For comparison with the case of the same but isolated crack ($l = 1$, $d = 0.25$), in Fig. 2a we give the narrow strip showing the function $\kappa^+(\omega)$ for this crack with the use of the same gray scale pattern. The values of the real parts of its three resonance poles closest to the real axis ($\operatorname{Re}\omega_n^1 = 1.33$, 2.11, and 2.34) are plotted as light dot-and-dash lines. The dark zones of considerable blocking of the isolated crack (bands $1.2 < \omega < 1.4$ and $2.1 < \omega < 2.3$) are located near these values.

In Fig. 2, attention is focused on the fact that frequency bands of blocking remain approximately the same as in the case of a single crack, although the positions of the poles ω_n vary with distance x_2. At first glance, this behavior contradicts the earlier inference [11, 12] about the role of resonance poles ω_n lying close to the real axis in the appearance of the blocking effect. However, a more careful analysis of the pole motions in the complex plane ω shows that there is no contradiction.

With increasing distance x_2, the values of $\operatorname{Re}\omega_n$ monotonically decrease and the negative imaginary parts $\operatorname{Im}\omega_n$ alternatively decrease and increase within certain limits with an upper boundary $\operatorname{Im}\omega_n = 0$. Thus, the poles ω_n move in the lower half-plane of the complex plane ω from left to right alternatively deviating

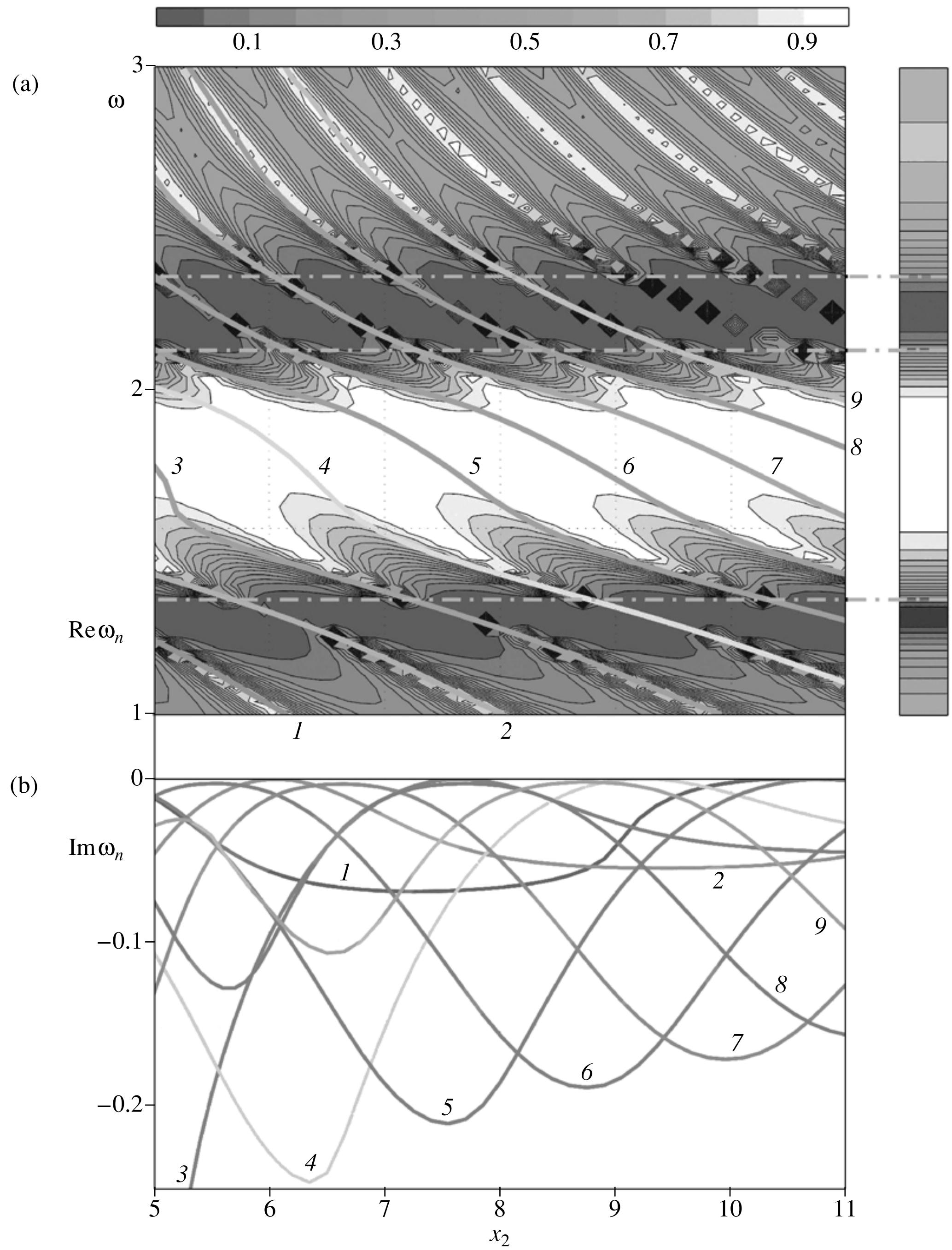

Fig. 2. Transmission coefficient $\kappa^{+}(x_2, \omega)$ and resonance poles $\omega_n(x_2)$ for two cracks at the depth $d = 0.25$ versus the distance x_2 between the cracks.

from and approaching the real axis up to the point of touching it at certain discrete values of x_2 (the appearance of the real point of discrete spectrum ω_n corresponds to the total energy blocking and the formation of undamped natural vibrations localized near the defect—see, e.g., Fig. 15 in [12]). The most interesting feature is that these motions are such that the poles approach the real axis only near the aforementioned frequency bands of blocking by a single crack and, when a pole leaves the axis, it immediately gives place to the next one that approaches the axis. As a result, nearly real-valued poles ω_n ensuring resonance blocking (the dark horizontal strips in Fig. 2a) are always present in these frequency bands irrespective of the distance between the cracks. The poles moving downwards rapidly cease affecting the blocking. For exam-

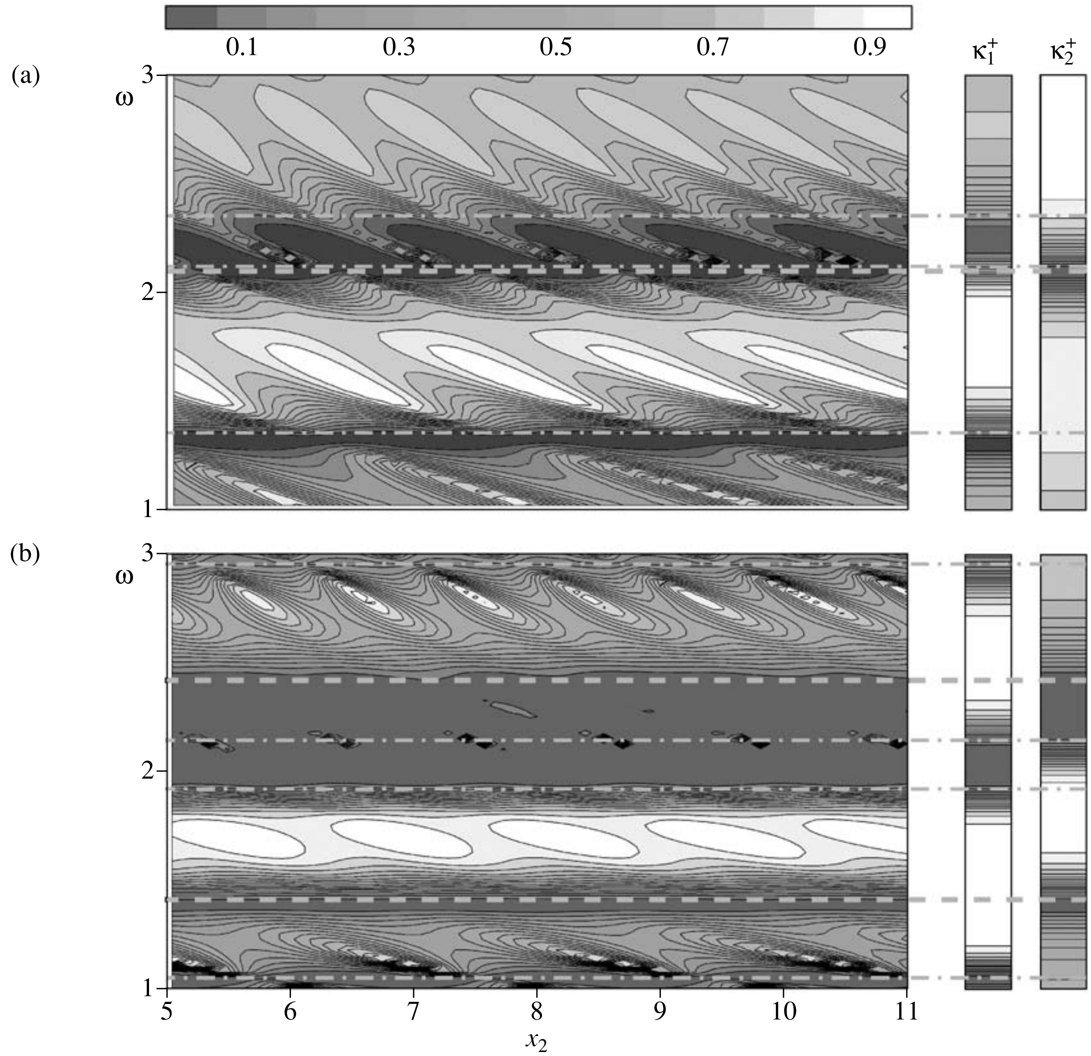

Fig. 3. Transmission coefficient $\kappa^+(x_2, \omega)$ for two cracks at different depths (a) $d_1 = 0.25$, $d_2 = 0.5$ and (b) $d_1 = 0.185$, $d_2 = 0.27$.

ple, only the poles with $\mathrm{Im}\,\omega_n < -0.1$ are present in the light strip of nearly total transmission $1.5 < \omega < 2$.

The analysis shows a less obvious and, therefore, more unexpected fact. Namely, at frequencies ω exactly coinciding with $\mathrm{Re}\,\omega_n$, the effect of blocking is rather decreased than increased, as in the case of a single crack. Light strips (transmission bands) traversing the dark or gray background along the curves $\mathrm{Re}\,\omega_n(x_2)$ are evidence of this. In the relief surface $\kappa^+(x_2, \omega)$, these transmission bands look like narrow walls (or mountain chains), whereas narrow depressions (canyons, as they were called in [11, 12]) were situated along $\mathrm{Re}\,\omega_n$ (for $|\mathrm{Im}\,\omega_n| \ll 1$) in the case of a single crack. With increasing $|\mathrm{Im}\,\omega_n|$, the chains become wider and lower and the canyons become wider and shallower and disappear (i.e., diffusing to the level of the surrounding surface) at approximately $|\mathrm{Im}\,\omega_n| > 0.15$–$0.2$. At low frequencies of the first blocking band $1 < \omega < 1.5$, the horizontal distance between the transmission curves (i.e., the distance between the curves $\mathrm{Re}\,\omega_n(x_2)$ at a fixed frequency ω) is approximately equal to half of the wavelength λ_0 of the zeroth antisymmetric mode a_0: $\lambda_0/2 = \pi/\zeta_1$, and the function $\kappa^+(x_2)$ is minimum in the middle of this distance.

In other words, resonance interaction between cracks occurs periodically (with a period of $\lambda_0/2$) with varying distance between them, and this interference manifests itself as deblocking of the waveguide. This process sharply changes the energy flux structure, in a way similar to that in the case of a single crack (see the description of Figs. 9 and 10 in [11]). The resonance blocking of the waveguide is accompanied by the appearance of strong energy vortices in the time-averaged energy flux of the field of steady-state harmonic vibrations $\mathbf{u}e^{-i\omega t}$. The system of vortices is formed above and below the crack, the maximum energy being concentrated in the rectangle between the crack and the nearest layer boundary (see Figs. 3 and 9 in [11]). When

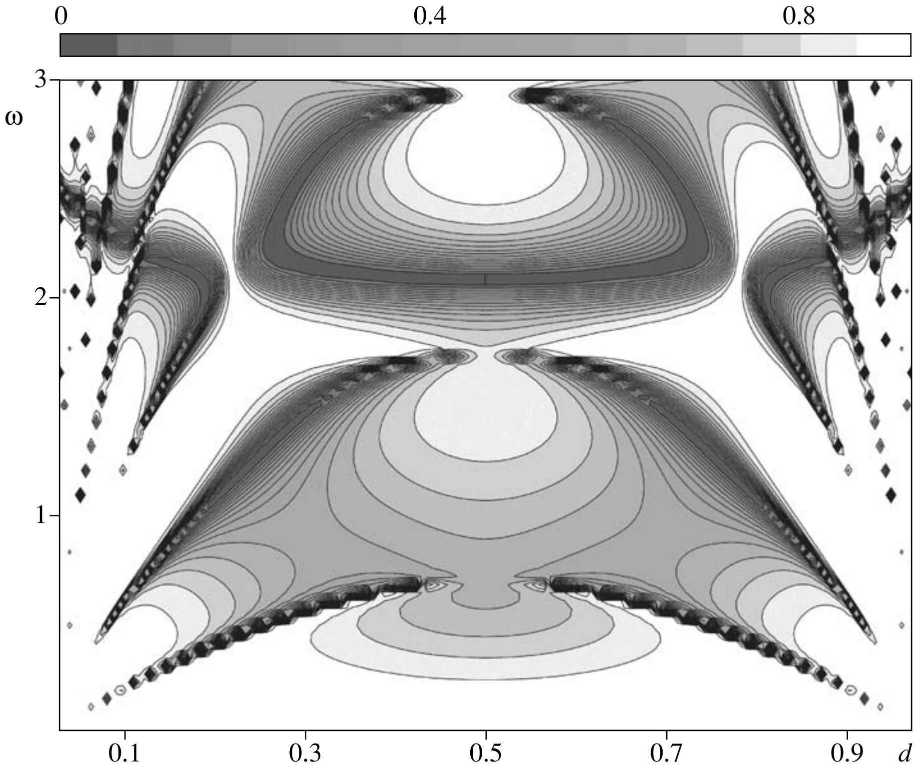

Fig. 4. Transmission coefficient $\kappa^+(x_2, \omega)$ for a single crack ($l = 1$).

the waveguide is deblocked, the system of vortices is replaced by a large single vortex, which encompasses the crack, and the energy flux rounds the obstacle along the periphery of this vortex and moves away to infinity ($\kappa^+ \approx 1$, see Fig. 10 in [11]). In the case of two cracks, a similar change in the energy flux structure occurs in the course of waveguide blocking and deblocking in the transmission band.

Spaced Cracks at Different Depths

As in the previous case, variation of the horizontal distance between the cracks does not change the frequency bands of blocking (see, e.g., Fig. 3 for the same cracks as those in Fig. 2, but at different depths (a) $d_1 = 0.25$, $d_2 = 0.5$ and (b) $d_1 = 0.185$, $d_2 = 0.27$). It is important to note that the blocking properties of the system of cracks are formed as the sum of the bands of blocking of every separate crack. These bands depend only on crack size and depth. Because of this, there is no need to solve the optimization problem for the whole system to calculate the geometry of a system of obstacles (cracks in our case) that would ensure the required blocking properties, such as vibration insulation in a given frequency range. It will suffice to examine the resonance blocking properties of a single obstacle as a function of its size and position. Then, the desired blocking parameters can be achieved by simple selection of a system of obstacles with complementary bands of blocking. Examples of such functions (at fixed size, depth, or frequency) are given in Figs. 6 and 7 of [11] and Figs. 10 and 11 of [12]. Figure 4 shows the pattern $\kappa^+(d, \omega)$ for the cracks of unit halfwidth ($l = 1$) as a function of depth and frequency; using this pattern, one can easily select the depths of several cracks to ensure signal filtration (or vibration insulation) in the required frequency range.

For example, the solid line in Fig. 5a shows the frequency-dependent transmission coefficient $\kappa^+(\omega)$ in the case of a waveguide with three cracks of unit halfwidth l ($d_1 = 0.185$, $d_2 = 0.27$, $d_3 = 0.29$; $x_2 = 9.5$, $x_3 = 20$), each of them ensuring resonance blocking at a certain frequency near $\omega = 2$ (the dashed and dot-and-dash lines), so as to ensure combined signal filtration in the band $2 < \omega < 2.5$. Blocking in a still wider frequency band is achieved with three cracks with $l = 2$ at depths $d_1 = 0.35$, $d_2 = 0.375$, and $d_3 = 0.4$; $x_2 = 20$ and $x_3 = 40$ (Fig. 5b).

Thus, although the mutual influence between cracks is perceptible at arbitrary distances (which, in particular, is evidenced by the variation of resonance pole positions in Fig. 2), the determination of blocking zones of a system of spaced cracks can be performed on the basis of the discrete approach, in which every individual crack is described by the reflection and transmission matrixes and the final coefficients are calculated by solving a system of linear equations.

Stacked Cracks

In the case of cracks not spaced along the waveguide, i.e., in the case when the projections of regions Ω_m on the x axis significantly overlap, each crack is actually located in a waveguide of smaller thickness. The transverse size of these waveguides is determined either by the distance from the layer boundary to the neighboring crack, or by the distance between the cracks located above and below the crack of interest (in the case of three-story cracks and higher). For this reason, estimation of blocking properties of the whole system on the basis of the results obtained for single cracks in a layer of unit thickness is limited and requires certain corrections. First of all, one should take into account the fact that resonance energy localization usually occurs in the rectangular area between the crack and the nearest surface of the layer or the half-space. As a result, a crack lying close to the waveguide surface or a neighboring crack of greater length will localize energy in this area at frequencies coinciding with those calculated for a single crack located at the same distance from the surface of the layer or the half-space (i.e., the effect of the opposite surface can be neglected). Second, in the case of validity of the assumption that every stacked crack behaves as a single crack located in the waveguide of a smaller thickness, the dimensionless frequencies of resonance blocking obtained for the layer of unit thickness can be easily recalculated to the case of the stacked cracks of interest.

For example, let two cracks with halfwidths of l_1 and l_2 be positioned one over the other in a waveguide of unit thickness ($H = 1$) at depths of d_1 and d_2 ($0 < d_1 < d_2 < 1$). The thicknesses of the imaginary waveguides for the first and second cracks are $H_1 = d_2$ and $H_2 = 1 - d_1$, respectively. We assume that results for a single crack of arbitrary halfwidth l located at arbitrary depth d in the layer of unit thickness are available, or they can be easily obtained. The dimensionless input parameters of the stacked cracks under consideration in the imaginary waveguides are $\tilde{l}_m = l_m/H_m$ and $\tilde{d}_m = d_m/H_m$, $m = 1, 2$. They define certain sets of dimensionless resonance frequencies $\tilde{\omega}_n^m$, $n = 1, 2, 3, \ldots$; $m = 1, 2$. Then, these frequencies are recalculated into the system of units related to the actual waveguide ($H = 1$) according to the relationships $\omega_n^m = \tilde{\omega}_n^m/H_m$.

Figure 6 gives an example of the frequency dependence of the transmission coefficient $\kappa^+(\omega)$ for two stacked cracks ($l_1 = l_2 = 1$, $d_1 = 0.25$, $d_2 = 0.85$) with blocking bands near the frequencies $\omega = 0.55$, 0.9, 1.35, 1.7, and 2.3. In this case, the thicknesses of the imaginary waveguides are $H_1 = 0.85$ and $H_2 = 0.75$, and the dimensionless parameters of the corresponding single cracks are $\tilde{l}_1 = 1.18$, $\tilde{d}_1 = 0.29$ and $\tilde{l}_2 = 1.33$, $\tilde{d}_2 = 0.8$. In Fig. 6, the dashed and dotted lines correspond to the transmission coefficients $\kappa_m^+(\omega)$ for each of the cracks alone versus the fundamental angular frequency $\omega = \omega^m/H_m$. It is seen that the above bands of resonance blocking by a system of stacked cracks is determined by the resonance properties of individual cracks.

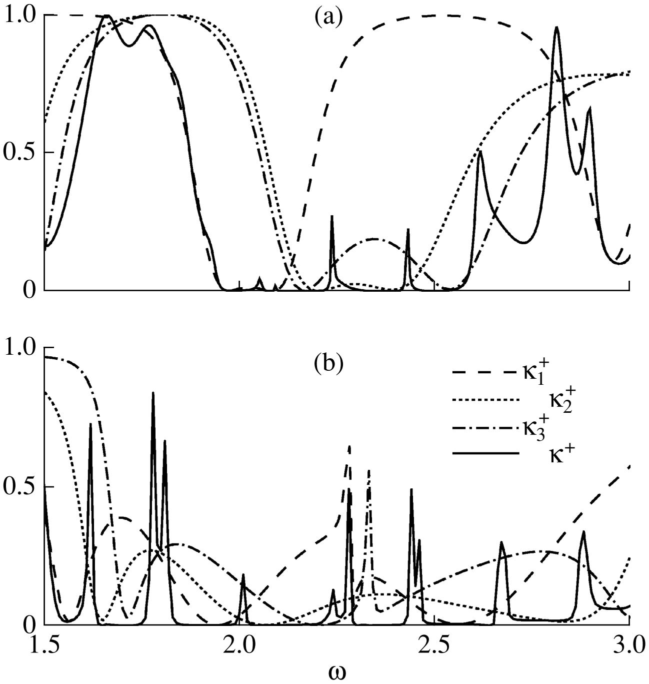

Fig. 5. Frequency dependence of the transmission coefficient κ^+ for the system of three cracks (the solid line) and for individual cracks (the dashed and dot-and-dash lines) for the crack length l = (a) 1 and (b) 2.

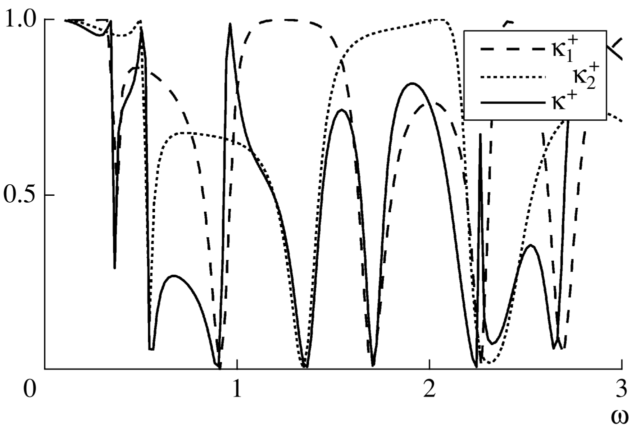

Fig. 6. Transmission coefficients $\kappa^+(\omega)$ for two story cracks (the solid line) and individual cracks in the corresponding waveguide of smaller thickness (the dashed and dotted lines).

CONCLUSIONS

On the basis of the numerical analysis carried out within the framework of the model of a two-dimensional elastic waveguide with several horizontal cuts (cracks), we found that, although the set of resonance poles ω_n cannot be obtained as a simple sum of poles ω_n^m corresponding to individual obstacles considered alone, the blocking properties of a system of cracks are often determined by the frequencies of trapped modes (frequencies of resonance blocking) of individual obstacles. Therefore, the results obtained for single cracks can be used to determine the parameters of an aperiodic system of several obstacles to ensure band gaps or vibration insulation in required frequency bands.

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REFERENCES

1. *Physical Acoustics: Principles and Methods*, Ed. by W. P. Mason (Academic, New York, 1968; Mir, Moscow, 1966), Vol. 1, Pt. A.
2. A. Figotin, Yu. A. Godin, and I. Vitebsky, Phys. Rev. **57**, 2841 (1998).
3. E. L. Ivchenko and A. N. Poddubnyĭ, Fiz. Tverd. Tela **48**, 540 (2006) [Phys. Solid State **48**, 581 (2006)].
4. J. O. Vasseury et al., J. Phys.: Condens. Matter **10**, 6051 (1998).
5. A. Avila, G. Griso, and B. Miara, C. R. Acad. Sci., Ser. I: Math. **340**, 933–938 (2005).
6. Ch. Zhang and D. Gross, *On Wave Propagation in Elastic Solids with Cracks* (Computational Mechanics Publ., Southampton, 1998).
7. C. M. Linton and D. V. Evans, Q. J. Mech. Appl. Math. **44**, 487 (1991).
8. Yu. I. Bobrovnitskiĭ and M. P. Korotkov, Akust. Zh. **37**, 872 (1991) [Sov. Phys. Acoust. **37**, 453 (1991)].
9. V. A. Babeshko, Izv. Vyssh. Uchebn. Zaved. Radiofiz. Sev.-Kavkaz. Region., Spec. Issue, 90–91 (1994).
10. D. A. Indeĭtsev and E. V. Osipova, Zh. Tekh. Fiz. **66** (8), 124 (1996) [Tech. Phys. **66**, 811 (1996)].
11. E. V. Glushkov, N. V. Glushkova, and M. V. Golub, Akust. Zh. **52**, 314 (2006) [Acoust. Phys. **52**, 259 (2006)].
12. E. Glushkov, N. Glushkova, M. Golub, and A. Boström, J. Acoust. Soc. Am. **119**, 3589 (2006).
13. E. V. Glushkov, N. V. Glushkova, and M. V. Golub, Prikl. Mat. Mekh. **71**, 702 (2007).
14. I. I. Vorovich, Dokl. Akad. Nauk SSSR **245**, 1076 (1979) [Sov. Phys. Dokl. **24**, 304 (1979)].
15. I. A. Viktorov, *Rayleigh and Lamb Waves: Physical Theory and Applications* (Nauka, Moscow, 1966; Plenum, New York, 1967).
16. P. D. Wilcox, M. J. S. Lowe, and P. Cawley, J. Intell. Mater. Syst. Struct. **12**, 553 (2001).

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